MODELING OF THE BEHAVIOR OF LOOSE MATERIALS IN A VIBRATING CHUTE

S. F. Yatsun and O. G. Maslova

UDC 621.867.522.2

A method is suggested for mathematical modeling of the behavior of loose materials, including variations of the volume concentration. The method is based on the assumption of the continuity hypothesis and equations of the mechanics of continua.

Vibration equipment and methods are widely used in the industry for loose material processing, for example, in grinding, classification, batching, transportation, mixing, drying, packing, etc.

Problems of theoretical and experimental studies of loose material behavior in a vibrating chute are treated in [1-7]. However, the models developed cannot be used to study the influence of the vibrator motion on the processed material.

In materials subjected to vibration, the structure and volume concentration continuously vary. Some of the processes are characterized by periodic thixotropic destruction. In other cases unsteady periodic thixotropy takes place. Therefore, mathematical modeling of the behavior of loose materials in a vibrating chute requires a model of loose materials both in structural and structureless states.

Let us assume that the material is isotropic and transition from one model to another depends on the level of the solid phase volume concentration which will be found from the formula

$$\nu = \rho/\rho_m.$$

We will restrict ourselves to such technological processes which involve only mass transfer and variation of the material density and which are adequately described by a plane computation scheme with an invariable size of the tank, isothermal conditions, and without chemical reactions. In this case, the flow processes can be described by the equations of mechanics of continua: the conservation laws of mass and momentum and a rheological equation which is based, on the one hand, on the formal methods of the mechanics of continua [8-9] and, on the other hand, on an analysis of the experimental data.

The restrictions enumerated above suggest that the stress tensor P depends only on the volume concentration ν and the strain rate tensor D:

$$P = P(\nu, D). \tag{1}$$

Series expansion of Eq. (1) with the linear term alone preserved gives

$$P = A_0(D_I, D_{II}, D_{III})I + A_1(D_I, D_{II}, D_{III})D. \tag{2}$$

Approximation of A_0 and A_1 by the linear functions of tensor D invariants yields after some manipulations:

$$P = (\alpha_0 + \alpha_0' D_I + \alpha_0'' D_{II} + \alpha_0''' D_{III})I + (\alpha_1 + \alpha_1' D_I + \alpha_1'' D_{II} + \alpha_1''' D_{III})D. \tag{3}$$

As applied to the plane stressed state, Eq. (3) takes the form

$$P = (\alpha_0 + \alpha_0' D_I + \alpha_0'' D_{II})I + (\alpha_1 + \alpha_1' D_I + \alpha_1'' D_{II})D, \tag{4}$$

Kursk Polytechnical Institute, Kursk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 63, No. 2, pp. 227-231, August, 1992. Original article submitted November 25, 1991.

1062-0125/92/6302-0832$12.50

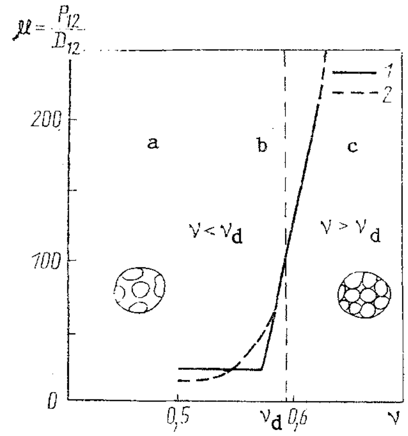

Fig. 1. Dependence of shear stresses at a constant shear rate on volume concentrations (quartz sand, 0.14-0.63-mm particle size): a) structureless system; b) dilatant jump; c) structured system; 1) theory; 2) experiment. $\mu = P_{12}/D_{12}$; N·sec/m^2.

where

$$D_I = D_{11} + D_{22}; \quad D_{II} = D_{11}D_{22} - D_{12}^2;$$

$$D = \begin{vmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{vmatrix}; \quad P = \begin{vmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{vmatrix}.$$

The present experiments with a shear flow and the results of [7], however, show that in a dense material flow the shear stress rapidly increases ten times, and if the stress level is lower than certain limited value, further shearing stops. Additional actions must be applied to the material for shearing to resume. The abrupt increase of the stress in a structured system may be ascribed to loose material dilatancy, which was first observed by Reynolds. Phase transitions in the material studied occur at a certain value of dilatant volume concentration ν_d and is called a dilatant jump.

In a rarefied flow no dilatancy is practically observed since the particles interact via impact impulse transfer.

In a dense material stream the shear flow takes place with direct contact of particles, resulting in transverse velocity components and normal shear stresses. The parameter ν_d depends on the grain material properties, particle diameter and form factor, and type of interparticle contacts and is found experimentally by a rotary viscometer which is used to study the shear flow of loose materials with a variable volume concentration.

The viscometer where compressed air can be supplied through the tank bottom allowed us to observe that, by increasing the air flow rate through the material and, consequently, changing the volume concentration from some initial value to ν_d, the dilatancy torque of the viscometer instantaneously rises, and if the drive power is small, the movable cylinder stops. By increasing the drive torque, it is possible to destroy a particle and to change the volume of bed adjacent to the movable cylinder of the viscometer and, consequently, to reduce the volume concentration to ν_d. Then shearing resumes. Figure 1 shows an example of a plot of shear stresses versus volume concentrations. Thus, ν_d is an important property of loose materials.

For adequate description of two qualitatively different states of loose material, determined by the volume concentration ν_d, using rheological equation (4), α_0, α_0', α_0'', α_1, α_1', α_1'' will be expressed as piecewise-constant functions of solid phase volume concentration that change their values at $\nu = \nu_d$. Due to the identification of parameters from the experimental results for quasistatic compression, viscometric flow, and vibroacoustic action, the parameters of these functions were determined in such a way that the rheological equation can adequately describe the material flow with a simple shear, quasistatic compression, plane stressed state.

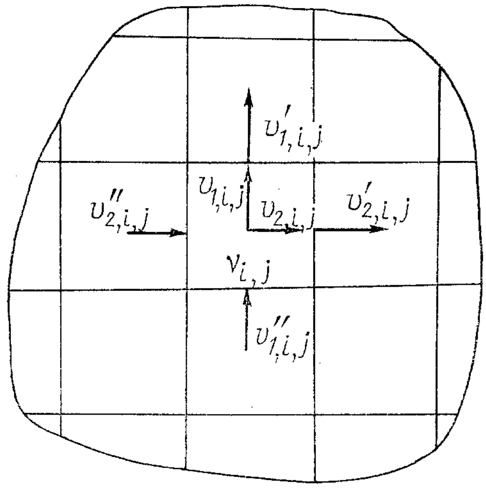

Fig. 2. Computation scheme.

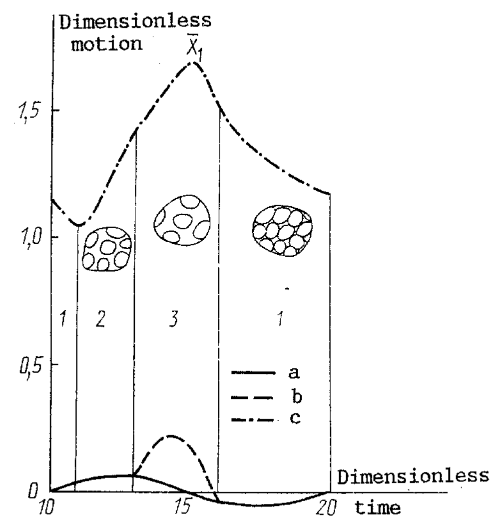

Fig. 3. Relationships of loose material motion in a vibrating chute: a) vibrating chute; b) lower boundary of material; c) upper boundary; zones: 1) compaction (structured system); 2) structure destruction; 3) flight (structureless system).

Thus, the system of differential equations describing a loose material flow on the $X_1 0 X_2$ plane with Eq. (4) has the form:

$$\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x_1} + v_2 \frac{\partial v_1}{\partial x_2} = \frac{1}{\nu \rho_0} \left(\frac{\partial P_{11}}{\partial x_1} + \frac{\partial P_{12}}{\partial x_2} \right) + g_1 + \xi_1 - R_1,$$

$$\frac{\partial v_2}{\partial t} + v_1 \frac{\partial v_2}{\partial x_1} + v_2 \frac{\partial v_2}{\partial x_2} = \frac{1}{\nu \rho_0} \left(\frac{\partial P_{12}}{\partial x_1} + \frac{\partial P_{22}}{\partial x_2} \right) + g_2 + \xi_2 - R_2, \quad (5)$$

$$\frac{\partial \nu}{\partial t} + \frac{\partial (\nu v_1)}{\partial x_1} + \frac{\partial (\nu v_2)}{\partial x_2} = 0;$$

$$P_{11} = P_0 + \alpha'_0 D_{\mathrm{I}} + \alpha''_0 D_{\mathrm{II}} + \alpha_1 D_{11} + \alpha'_1 D_{11} D_{\mathrm{I}} + \alpha''_1 D_{11} D_{\mathrm{II}},$$

$$P_{22} = P_0 + \alpha'_0 D_{\mathrm{I}} + \alpha''_0 D_{\mathrm{II}} + \alpha_1 D_{22} + \alpha'_1 D_{22} D_{\mathrm{I}} + \alpha''_1 D_{22} D_{\mathrm{II}}, \quad (6)$$

$$P_{12} = \alpha_1 D_{12} + \alpha'_1 D_{\mathrm{I}} D_{12} + \alpha''_1 D_{\mathrm{II}} D_{12}.$$

If $\nu > \nu_d$, $D_{12} = 0$ and $\partial P_{12}/\partial x_i < (\partial P_{i,i}/\partial x_i) f_{lim}$ ($i = 1, 2$), then $D_{12} = 0$. Here Eq. (5) is the equation of momentum and mass conservation, Eq. (6) is the rheological equation.

For a description of the aerodynamic drag, we will use the model suggested in [5].

In the problems of vibration technology with variable inertial forces exerted on the material and continuous variation of the volume concentration, the system of equations (5), (6) can be only obtained by numerical methods.

Analysis of various numerical methods and preliminary calculations have shown that the most favorable method that ensures stable count is the large particle technique developed by O. M. Belotserkovskii and Yu. M. Davydov [10].

The method consists in splitting the initial system of Euler's unsteady equations, written as the conservation laws, between the physical processes. A steady-state solution of the problem, if any, can be reached by calculating the transient regime of the system from the initial conditions, repeating the computation many times in time. This method is implemented with a three-step algorithm, according to which the equation set is, first, integrated with the convective terms on the Lagrangian net neglected. At the second step, the amount of material transferred from one cell to another is determined, and at the third step the velocity in the cells is recalculated with the mass transfer process included.

Thus, the material modeled consists of $N \times M$ particles, which are distributed on an Eulerian net at the initial time in accordance with the initial conditions. The motion of such a system of particles for the time Δt is realized, first, as a variation of their intrinsic state, assuming they are immovable; then the computation net is recalculated for the initial state. The initial debugging of the algorithm was performed in the problems of a falling liquid column, a water flow through a sluice, whose solution was obtained by Harlow with the net marker method [11]. This stage revealed rational net parameters and gave the time step.

During modeling of the loose material flow we have found that as the material density increases in some of the cells and when the shear stresses are low, the condition $D_{12} = 0$, corresponding to the absence of shear at a given point, started operating. However, due to the viscous effects of the computation scheme in the cells shearing did not stop immediately, and the density continued to rise. To eliminate this effect the integration procedure was modified. On the one hand, it is based on the ideas of a variable integration step that diminishes at $\nu > \nu_d$, and on the other hand, on updating of the finite-difference condition of the absence of shear on a phenomenological basis.

In the former case, the computation rate decreases substantially; the second option, which includes additional limitation, appeared more useful. The following constraint was introduced: as soon as the density in a cell becomes ν_d, velocities at its boundaries are assumed to be zero (Fig. 2)

$$v'_{1,i,j} = v''_{1,i,j} = v'_{2,i,j} = v''_{2,i,j} = 0,$$

and the material supply to the cell stops. Because of this, the material was stopped at $\nu = \nu_d$, actually at the subsequent integration step.

Preliminary computations have revealed a domain of parameters where stable count with a relatively high accuracy is maintained. Calculation of loose material motion through a horizontal chute, vibrating at a low frequency, is given as an example (Fig. 3). Figure 3 shows one period of the external harmonic action. The results were verified by high-speed photography of the behavior of a loose material subjected to vibration in a transparent vessel.

The method developed may be used for studying the behavior of loose materials subjected to vibration widely used in industry, for determination of the instantaneous and vibration period-averaged densities and the density at any point along the bed height, instantaneous and average velocities of vibrotransportation, and power characteristics of the vibrator.

NOTATION

ν, solid phase volume concentration; t, time; ρ, material density; ρ_m, grain material density; D, strain rate tensor; D_I, D_{II}, D_{III}, tensor invariants; I, unit matrix; α_0, α_0', α_0'', α_0''' α_1, α_1', α_1'', α_1''' solid phase volume concentration functions; OX_1, OX_2, axes of the relative coordinate system; v_1, v_2, relative velocity projections; $\ddot{\xi}_1$, $\ddot{\xi}_2$, g_1, g_2, projections of vibration acceleration and free fall acceleration; P, stress tensor; P_{11}, P_{12}, P_{22}, stress tensor components; D_{11}, D_{12}, D_{22}, components of the strain rate tensor; R_1, R_2, projections of specific force of aerodynamic drag; P_0, equilibrium stress; f_{lim}, limited coupling coefficient; $v'_{1,i,j}$, $v''_{1,i,j}$, $v'_{2,i,j}$, $v''_{2,i,j}$, velocity at the boundaries of the i-th and j-th cells.

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